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ORIGINAL ARTICLE

Population projections in small areas combining the forecast error of a time series method with the linear growth trend method (AiBi)*

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The 2022 Brazilian Demographic Census underscores an advanced stage of demographic transition, characterized by sustained fertility decline, shifting age structures, and a marked deceleration in population growth. Between 2010 and 2022, numerous municipalities experienced significant growth attenuation, with several exhibiting population decline. These demographic shifts, compounded by persistent socioeconomic and climatic disparities, have intensified the demand for robust subnational population projections. In Brazil, the AiBi method is widely employed however; its predictive accuracy has not been systematically evaluated. This study aims to quantify uncertainty in municipal population projections by sex, integrating a Random Walk with Drift method with AiBi's point projections. The proposed framework was applied to municipalities in the states of Rio Grande do Norte and São Paulo, regions with contrasting data quality and socioeconomic profiles. Our findings demonstrate that uncertainty is a function of population size, historical trend stability, and the projection horizon. Specifically, uncertainty intervals widen in smaller municipalities, in jurisdictions with volatile growth patterns, and as the forecast horizon extends.

Keyword: Projections. Small areas. AiBi. Random walk. Prediction interval.

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Introduction

The demand for subnational population projections – essential for strategic planning across economic, political, social, and environmental sectors in both public and private spheres – has intensified substantially in Brazil. Within the realm of social planning, there is a burgeoning requirement to integrate demographic dynamics into programmatic design, policy formulation, and evidence-based decision-making for resource allocation. This is particularly critical given the escalating pressure on public services such as healthcare, labor markets, education, sanitation, and social security. From this perspective, achieving a reliable degree of forecasting precision regarding the future of populations' size of small areas is indispensable for the effective design and implementation of public policy.

Estimating future population sizes – whether over short or long-term horizons and at granular levels of disaggregation – is only attainable through population projections. The adoption of methodologies capable of incorporating stochastic uncertainty into these estimates is essential, as projections for smaller geographic units inherently possess higher degrees of variability and predictive error.

Brazil, however, is characterized by profound socioeconomic and regional heterogeneities (World Bank, 2003; Bucciferro; Ferreira de Souza, 2020; Martins, 2023). Within the same national territory, populations residing in highly developed, economically dynamic clusters coexist with others – often in immediate proximity – defined by stunted development and low productivity (World Bank, 2003). Furthermore, Brazilian regions exhibit significant variability in wealth distribution, climatic conditions, and geographic scale. It is, therefore, plausible to posit that these regional disparities become increasingly acute as the scale of analysis shifts toward finer subnational geographic levels.

Since the mid-20th century, Brazil's demographic regime has undergone profound structural shifts across its primary components, manifesting with significant temporal and spatial heterogeneity (Queiroz *et al.*, 2020; Alves *et al.*, 2010; Brito, 2007). While this trend is well-documented for major administrative regions and federal units, it is particularly salient when analyzing smaller geographic scales. In these areas, small-population constraints heighten the complexity of applying standard demographic techniques. Furthermore, preliminary findings from the 2022 Demographic Census corroborate that Brazil and its constituent regions have reached an advanced stage of the demographic transition (IBGE, 2023).

Against this backdrop, there has been an increasing emphasis on developing methodologies and techniques that facilitate probabilistic forecasting of the future population size. Furthermore, the nature, quality, and granularity of available data present significant hurdles for demographic projections – particularly when conducted at the municipal or intra-municipal levels. Consequently, a diverse array of techniques for small-area population projections has emerged, each varying in its underlying assumptions, methodological sophistication, and data requirements (Waldvogel, 1998; Rao, 2003; Smith; Tayman; Swanson, 2001; Jannuzzi, 2006).

Regarding the consistency and precision of population estimates, a key consideration is the magnitude of the estimation error relative to the true, albeit latent, parameter. Population projections for expansive jurisdictions or high-density regions are typically predicated on demographic equilibrium shifts observed in the recent past. This equilibrium is governed by fluctuations in the primary demographic components – fertility, mortality, and migration – each of which is independently estimated and projected. Although modeling the current and future trajectories of these rates are a rigorous undertaking – requiring a robust understanding of the demographic transition theories that inform scenario-building – the outputs are generally deemed satisfactory. This is largely attributable to substantive improvements in census data quality and vital registration systems over recent decades (Queiroz *et al.*, 2017; Szwarcwald *et al.*, 2019).

Conversely, estimating age-specific mortality, fertility, and migration for small areas – particularly at municipal or intra-municipal levels – presents significant challenges. These difficulties arise from limited exposure to risk (due to small populations' size), heightened stochastic fluctuations, and substandard data quality, specifically regarding age misreporting and coverage errors (Schmertmann *et al.*, 2013, 2024). Consequently, the precision of a population forecast adheres to a fundamental axiom in demography: the smaller the geographic unit and the more distant the projection horizon, the greater the uncertainty inherent in the estimate.

Statistical theory, underpinned by the Law of Large Numbers, demonstrates that variability is inherently higher in smaller datasets or limited trial sets (Lohr, 1999). For instance, observing 100% frequency of "heads" in a three-coin-toss sequence is statistically plausible; however, as the number of trials scales to one thousand, the distribution predictably converges toward equilibrium of 50% heads and 50% tails. Applying this principle to demography, population estimates for small municipalities necessitate rigorous scrutiny: small-population constraints induce heightened stochastic variability, where fluctuations may arise purely by chance rather than reflecting underlying structural trends (Justino *et al.*, 2012).

The heightened uncertainty inherent in small-area estimates underscores the imperative for population projections that incorporate prediction intervals, especially within low-density jurisdictions. Population size serves as a pivotal determinant in several dimensions of public policy formulation, most notably in fiscal resource allocation. Queiroz *et al.* (2026) contend that absolute population counts should not constitute the sole criterion for the distribution of the Municipal Participation Fund. This is particularly critical for smaller municipalities, where population figures are susceptible to more substantial stochastic fluctuations, potentially undermining the equitable allocation of public funds.

In this context, small-area population projections must account for both estimation errors – contingent upon the geographic scale and population size – and forecast errors, which are intrinsically linked to the projection horizon. Furthermore, projections for small areas must ensure internal consistency; the aggregate populations of smaller areas must be reconciled with the total projected population of the overarching administrative region to which they belong (Waldvogel, 1998).

Given this background, this article proposes a methodological framework that integrates small-area population projection techniques with time-series forecasting. This framework enables the estimation of uncertainty, ensuring that future population size projections for small areas in Brazil are accompanied by robust prediction intervals. Consequently, the approach provides a measure of forecast variability and precision at each time point while maintaining internal consistency – ensuring that the sum of municipal projections aligns with the total projected for the respective higher-level administrative unit. The proposed exercise synthesizes the traditional deterministic point-projection method (AiBi) with the forecast error derived from a Random Walk with Drift (RWD) model. Our findings demonstrate that this integrated approach is flexible, computationally efficient, and constitutes a viable alternative for generating both point and interval population projections in small-area contexts.

Data

The data requirements for implementing the proposed technique are straightforward and readily accessible. Initially, the method utilizes a time series of total population estimates for small areas up to the most recent census, followed by population projections for the aggregate-level area over a predefined forecast horizon.

In the Brazilian context, the small-area units correspond to municipal-level populations disaggregated by sex, while the parent jurisdictions comprise state-level estimates and projections, similarly disaggregated by sex.

The decision to project the population by sex is predicated on the distinct growth dynamics inherent to each group. Male and female mortality trajectories diverge significantly in both magnitude and age-specific patterns. Furthermore, in small-area contexts, migration exerts a more pronounced influence on population dynamics; consequently, if migratory flows exhibit sex-specific differentials, modeling these subpopulations independently is essential to capture the underlying demographic drivers accurately.

To evaluate the proposed methodology, we selected two states from distinct geographic regions – Rio Grande do Norte and São Paulo – which exhibit contrasting socioeconomic, demographic, and regional profiles, as well as varying levels of data quality. Consequently, empirical tests were conducted using the municipalities (small-area units) nested within these two states (aggregate-level jurisdictions).

For both Rio Grande do Norte and São Paulo, the dataset is derived from the official population projections produced by the Brazilian Institute of Geography and Statistics (IBGE), specifically the 2024 revision (IBGE, 2024b). Regarding the municipalities in both states, the study utilizes the annual series of municipal population estimates, which IBGE has submitted to the Brazilian Federal Court of Accounts (TCU in Portuguese) since 1989.

The IBGE municipal series is consistently calculated using the AiBi method, detailed in the subsequent section. The projection parameters for each municipality are estimated based on data from the two most recent censuses. Consequently, between 2011 and 2021, the municipal population estimates produced for the TCU relied on AiBi parameters derived from the 2000 and 2010 censuses, without incorporating data from the 2022 demographic round. As a result, any shifts in population growth occurring between 2010 and 2022 remain uncaptured by the municipal series provided to the TCU. Following the same logic, the municipal population estimates were based on extrapolations from the 1991 and 2000 census cycles.

To account for potential shifts in population growth between terminal censuses years of each decade (2000-2010 and 2010-2022), we reconstructed annual series of municipal population estimates using the AiBi method. Specifically, we performed back projections grounded in the two censuses benchmarks that delimit each period. In a departure from IBGE's standard approach, our 2010-2022 annual series was estimated using the most recent censuses pair (2010 and 2022); a consistent procedure was applied to the 2000-2010 intercensal years. Consequently, we re-estimated the annual municipal populations from 2000 to 2022, yielding a harmonized time series that served as the input for the random walk with drift model detailed below. For the aggregate levels (the states of Rio Grande do Norte and São Paulo), we retained IBGE's official state-level population projections from the 2024 revision.

It is essential to underscore that prior to generating the intercensal population estimates, the municipal census populations' counts for 2000, 2010, and 2022 were adjusted for census under enumeration. As IBGE's Post-Enumeration Surveys (PES) does not yield municipality-specific omission rates, municipal census populations were corrected using state-level omission factors[1] disaggregated by age and sex, as provided in the IBGE 2024 revision. This procedure rests on the assumption of spatial homogeneity in undercount levels across all municipalities within a given state. While this assumption may not be universally applicable, it does not substantially undermine our projections, as the aggregated corrected municipal populations remain internally consistent with the state-level totals throughout the 2000-2022 period.

[1] The correction factor is derived from the IBGE state-level projections as provided in the 2024 revision (IBGE, 2024). Specifically, for each age group and sex in each census year (2000, 2010, and 2022), a state-level adjustment factor was calculated as the ratio between the IBGE-projected population (2024 revision) and the corresponding enumerated population from that year's census.

Consequently, the adjusted municipal census population is obtained by multiplying the enumerated population for a given year (2000, 2010, or 2022) by its respective state-level correction factor, as detailed above. This adjusted figure serves as the base population for constructing the historical series via the AiBi method for the 2000-2022 period.

Methods

The AiBi method: constant growth apportionment

Among the various population growth distribution methods – alternatively categorized as shift-share techniques – the framework proposed by Pickard (1967) and introduced to the Brazilian context by Madeira and Simões (1972) maintains significant prominence. Although this approach appears under diverse nomenclatures within the demographic literature (González; Torres, 2012; Santos; Barbieri, 2015; Grupo de Foz, 2021), it is most ubiquitously recognized in Brazil as the AiBi method.

The AiBi operates on the assumption that small-area units are nested within a broader aggregate area, for which total population projections have been previously established – ideally utilizing the cohort-component method or an equivalent demographic variant. As detailed in Grupo de Foz (2021), the total population of the aggregate area at time t is partitioned into n constituent units. Within this hierarchical structure, the population of the aggregate area is defined as the summation of the populations of all constituent small areas at that specific point in time:

$$P(t) = \sum_{i=1}^{n} P_i(t) \qquad (1)$$

where $P(t)$ denotes the total population of the aggregate area at time t, $P_i(t)$ represents the population of the *i-th* constituent unit at the same temporal point, and n signifies the total number of small areas nested within the aggregate jurisdiction.

Assuming that, throughout the projection horizon, the population of each constituent unit varies as a linear function of the aggregate area's population, this relationship can be formally expressed as:

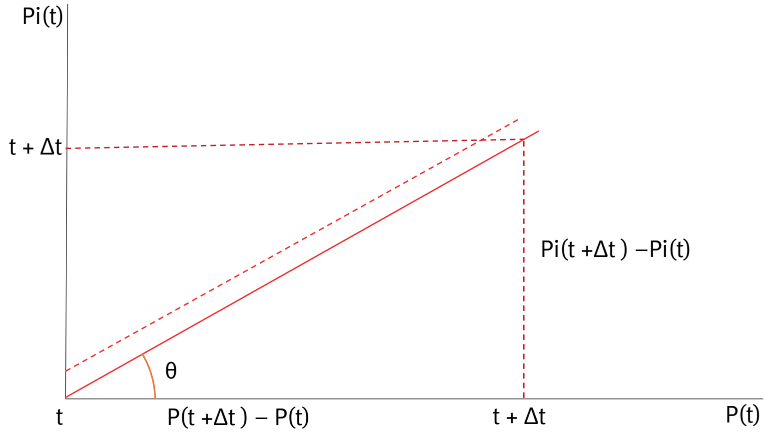
$$P_i(t) = b_i + a_i P(t) \qquad (2)$$

where a_i denotes the proportionality coefficient, representing the incremental change in the population of small area i relative to that of the aggregate area, and b_i serves as the linear correction coefficient (or intercept). This linear dependence between the population increments of the aggregate and constituent areas, as formulated in equation (2), is depicted in Figure 1.

The slope of the line depicted in Figure 1 represents the ratio of the population change in the constituent unit to that of the aggregate area between two consecutive censuses cycles. This parameter, denoted as a_i in equation (2), is determined as follows:

$$a_i = \frac{P_i(t+\Delta t) - P_i(t)}{P(t+\Delta t) - P(t)} \qquad (3)$$

FIGURE 1
Schematic representation of the linear relationship formulated in equation (2)



Source: Authors' elaboration.

Since this linear relationship does not necessarily intersect the origin, a level adjustment (or y-axis intercept) is incorporated via the coefficient b_i, which is determined as follows:

$$b_i = P_i(t + \Delta t) - a_i P(t + \Delta t) \quad (4)$$

Once the a_i and b_i coefficients are derived from the population data from the two most recent censuses cycles, equation (2) can be utilized to estimate the population of each constituent unit *i* for any given forecast horizon – or for retrospective estimation – contingent upon the availability of a population projection for the aggregate area at that same temporal point.

Based on the preceding derivation, the AiBi method is predicated on two fundamental assumptions:

1. **Linear Functional Form:** throughout the projection horizon, the population of each constituent unit is assumed to vary as a linear function of the aggregate area's population.
2. **Systemic Internal Consistency:** to ensure the framework's mathematical coherence, the summation of the proportionality coefficients across all constituent units must equal 1, while the summation of the linear correction coefficients must be equal 0. This constraint secures a primary advantage of the method: the aggregate of the small-area projections is inherently reconciled with the total projected population of the overarching region.

Despite its advantages, the AiBi method is subject to three critical limitations that warrant consideration:

1. **Divergent Growth Trajectories:** the framework exhibits mathematical inconsistency when constituent units experience population growth in opposing directions (e.g., simultaneous expansion in some areas and degrowth in others).

2. **Sensitivity to Trend Reversals:** the method yields implausible results if the population growth trajectory of the aggregate area undergoes a directional shift – transitioning from positive to negative growth, or vice-versa.
3. **Parameter Indeterminacy:** because the method's parameters are derived through bipunctual estimation (relying on data from only two temporal points), the framework inherently lacks a mechanism to quantify the statistical error or uncertainty associated with these parameters.

The first two limitations can be mitigated by stratifying the constituent units into two distinct groups – those exhibiting population growth and those undergoing decline – based on the two most recent census benchmarks. Within this framework, it is imperative to establish a partitioned projected value for the aggregate area, as the sum of the populations in municipalities experiencing growth will not, by definition, correspond to the total state population. Likewise, the aggregate population of municipalities in decline will not equal the state-level total, necessitating a dual-track reconciliation of these sub-aggregates.

Accordingly, the aggregated areas is bifurcated into two distinct sub-aggregates: (i) the segment comprising municipalities exhibiting population growth, and (ii) the segment encompassing the combined populations of municipalities that experienced population degrowth between the two most recent censuses benchmarks.

The population for these sub-aggregates throughout the projection horizon was estimated according to the following protocol:

- **Geometric Growth Calibration:** the geometric growth rate for the 2010-2022 intercensal period was calculated specifically for the sub-aggregate comprising municipalities with declining populations;
- **Targeted Projection**: utilizing this rate, the population for this declining segment was projected annually across the forecast horizon. This was executed via a geometric growth model, using the observed 2022 Census population as the baseline;
- **Residual Derivation**: given the availability of official state-level projections from Brazilian Institute of Geography and Statistics (IBGE), the population for the remaining sub-aggregated – consisting of municipalities exhibiting growth – was derived residually. This was achieved by subtracting the projected population of the declining-area group from the total projected state population.

Despite the need for these adjustments, the method's widespread application – including its use in the production of official population estimates for Brazilian municipalities by IBGE (IBGE, 2024a) – suggests that its advantages outweigh these initial limitations.

To address the third limitation, we propose an integration of the AiBi method with a stochastic component – specifically, the forecast error derived from a Random Walk with Drift model. This synthesis facilitates the generation of population projections accompanied by probabilistic uncertainty intervals, thereby providing a more robust and informative framework for small-area forecasting.

Random Walk with Linear Trend Model

The Random Walk with Drift model belongs to the class of forecasting frameworks characterized by a general trend component (Hamilton, 1994; Brockwell; Davis, 2016). Its defining characteristic lies in the integration of a deterministic linear trend with a stochastic random walk component. This model can be expressed as:

$$Y_t = \mathrm{c} + Y_{t-1} + e_t \tag{5}$$

where:

Y_t denotes the realization of the time series (in this context, the population) at time t;
c represents the drift parameter, accounting for the constant linear trend;
t signifies the temporal index of the series;
e_t represents the stochastic term (white noise), assumed to be independently and identically distributed (i.i.d.) following a Normal Distribution with mean of zero and constant variance σ^2.

Consequently, a Random Walk with Drift constitutes a parsimonious statistical framework or modeling time series that undergo stochastic fluctuations while maintaining a systematic upward or downward trajectory. When $c = 0$, the process fluctuates randomly around its preceding value, exhibiting no long-term trend. Conversely, if $c > 0$, the process follows positive deterministic drift, characterized by a growth trend punctuated by random disturbances. If $c < 0$, the series exhibits a systematic downward trajectory over time (Brockwell; Davis, 2016).

When generating projections based on this framework, the forecast uncertainty (or forecast error) is derived from the integration of two distinct components:

$$E_h = \sqrt{(h\sigma^2) + (he_p)^2} \tag{6}$$

where:

E_h denotes the forecast error at an h-steps-ahead horizon;
h represents the forecast lead time (number of periods ahead);
$h\sigma^2$ signifies the accumulated stochastic variance of the random walk component. The process accumulates variance as the forecast horizon (h) expands; specifically, the variance over h future periods is proportional to $h\sigma^2$, reflecting the temporal aggregation of the white noise terms (e_t);
$(he_p)^2$ represents the accumulated variance arising from the estimation of the linear trend (drift). The drift coefficient c is estimated from historical data, and this estimate carries an associated uncertainty (denoted by the standard error, $e_{p,}$). The additional error at h future steps, stemming from this uncertainty, scales with h^2, as the impact of the trend component compounds over time.

In short-term, the forecasts (small h), predictive uncertainty is dominated by the stochastic noise component σ^2. Conversely, in long-term horizons (large h), the uncertainty contribution from the trend e_p scales quadratically and may eventually supersede that of the random noise. For further theoretical foundations and practical applications of time series modeling, see Ferreira (2018).

Forecast error of the random walk with drift model integrated with AiBi point projections

Before detailing our proposed methodology – which synthesizes the stochastic uncertainty of the random walk model (E_h) with the point estimates generated by the AiBi method – it is pertinent to address a fundamental question: why not project municipal populations directly using a time series framework that inherently provides forecast uncertainty, thereby bypassing the AiBi method altogether?

As previously noted in the introduction section, small-area population projections must ensure internal consistency between the summation of municipal populations and the total population of the overarching jurisdiction. In other words, the small-area projections must be reconciled with the independent projection of the larger area, which serves as a pre-established constraint (Waldvogel, 1998). Henceforth, we will refer to this prior larger-area projection as the independent projection.

To illustrate these dynamics, Table 1 compares three distinct projection approaches for female population in the two states analyzed in this study: state-level female population derived from IBGE 2024 revision[2] (IBGE, 2024b); state-level female population obtained by aggregating municipal projections through the traditional AiBi method; and state-level female population resulting from the summation of individual municipal projections generated solely via the random walk with drift model.

TABLE 1
Comparison of state-level female population projections
Rio Grande do Norte and São Paulo – 2022-2070

Source	2030		2040		2050		2060		2070	
	RN	SP	RN	SP	RN	SP	RN	SP	RN	SP
IBGE, 2024 (1)	1,793,185	24,013,307	1,807,496	24,100,574	1,777,266	23,592,304	1,696,982	22,484,360	1,576,978	20,929,646
AiBi	1,793,185	24,013,307	1,807,496	24,100,574	1,777,266	23,592,304	1,696,982	22,484,360	1,576,978	20,929,646
Random walk	1,868,541	25,069,633	2,006,210	26,933,346	2,143,879	28,797,059	2,281,549	30,660,771	2,419,218	32,524,484
dif (%)	4.20	4.40	10.99	11.75	20.63	22.06	34.45	36.36	53.41	55.40

Source: IBGE (2024b); IBGE Demographic Censuses 2000, 2010 and 2022.
(1) Independent projection, that is, the female population projected by IBGE in its 2024 revision.

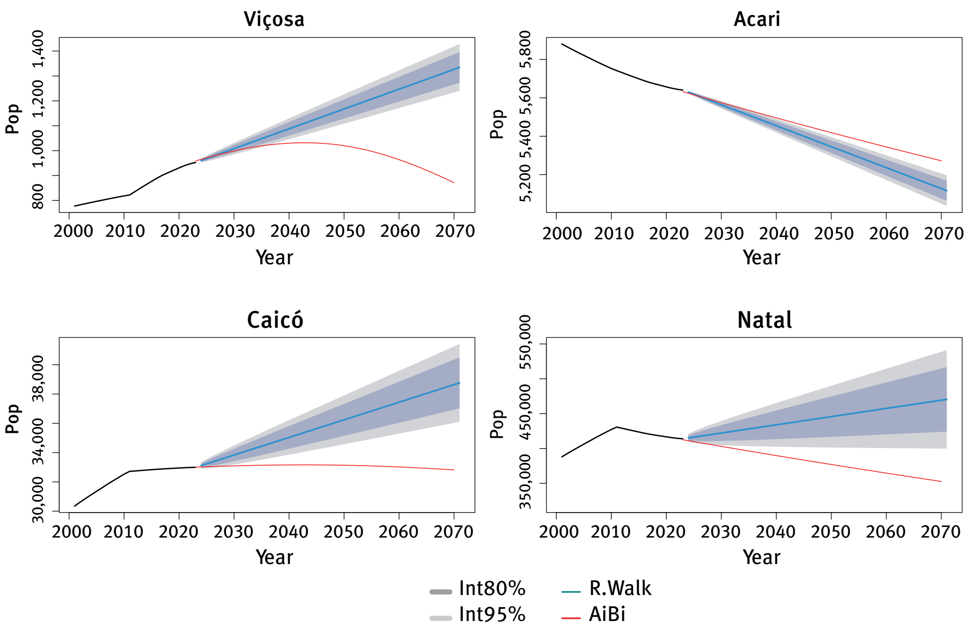
[2] These estimates constitute the independent state-level projections for Brazil, as produced by IBGE (2024b). This dataset provides age- and sex-specific projections for all Brazilian federative units, derived via the cohort-component method. In the present analysis, we aggregate the female population across all age cohorts to establish a control total or the projected female population. This aggregate serves as the benchmark against which the summation of municipal-level female population projections is evaluated for each respective state.

The data presented in Table 1 demonstrate that the aggregated municipal projections generated via the AiBi method align perfectly with IBGE state-level reference (independent projection). Conversely, the summation of municipal projections derived from the random walk with drift model fails to reconcile with the official state totals, exhibiting a clear numerical discrepancy.

The bottom line of Table 1 illustrates the relative percentage deviation between the aggregated municipal projections (random walk model) and the independent state-level projection by IBGE. These discrepancies diverge from the forecast horizon because each municipal projection in the random walk framework is executed autonomously, relying exclusively on localized historical trends without accounting for state-level dynamics. In contrast, the AiBi method imposes a structural constraint on local projections, ensuring they remain congruent with the aggregate population of the overarching area, thereby preserving top-down coherence.

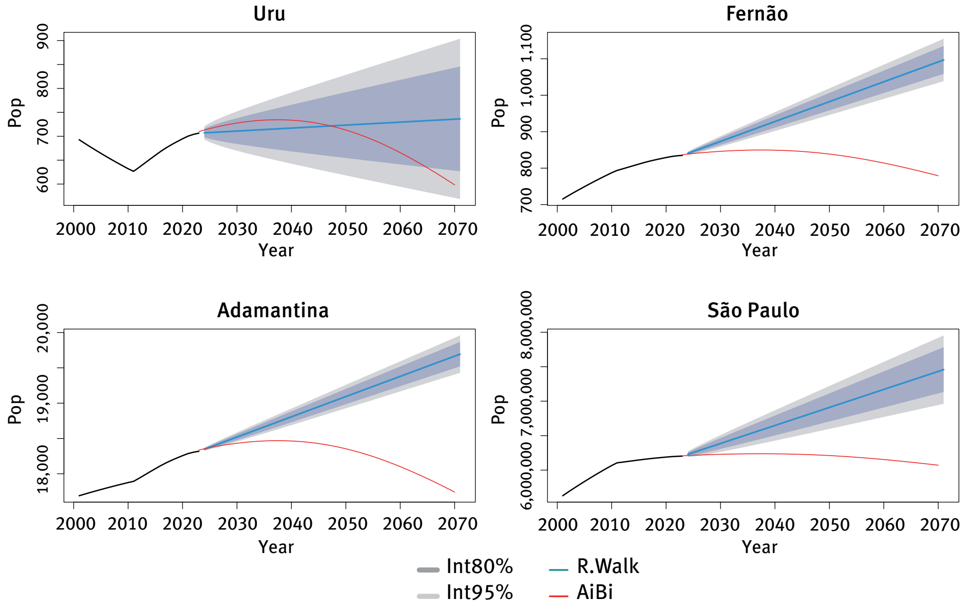
Figures 2 and 3 illustrate these disparities by overlaying the female population projections derived from the random walk with drift model – including 80% and 95% confidence intervals – with the AiBi point projections. This comparison is presented for four selected municipalities within each of the two analyzed states, juxtaposing the stochastic uncertainty of the time series model against the deterministic consistency of the AiBi framework.

FIGURE 2
Historical observed series (2000-2022) and female population projections (2023-2070): Integration of random walk with drift (including 80% and 95% confidence intervals) and AiBi point projections
Selected municipalities in Rio Grande do Norte – 2000-2070



Source: IBGE. Brazilian Demographic Censuses 2000, 2010, and 2022, adjusted using IBGE state-level projections, Revision 2024 (IBGE 2024b).

FIGURE 3
historical observed series (2000-2022) and female population projections (2023-2070): Integration of random walk with drift (including 80% and 95% confidence intervals) and AiBi point projections
Selected municipalities in São Paulo – 2000-2070



Source: IBGE. Brazilian Demographic Censuses 2000, 2010, and 2022, adjusted using IBGE state-level projections, Revision 2024 (IBGE, 2024b).

These figures reveal that the AiBi projections follow a trajectory distinct from those generated by the random walk with drift model. The time series method extrapolates historical trends into the future without external constraints, as it fails to account for the aggregate state-level population. Consequently, municipal projections under this unconstrained model may exhibit implausible growth or decline over the forecast horizon, diverging from demographically realistic expectations.

While the AiBi method similarly extrapolates historical trends, it operates within the structural boundaries imposed by the population trajectory of the overarching area, as formally defined in equations (2), (3), and (4). This ensures that localized projections remain mathematically anchored to the broader demographic trend of the state.

Both methodologies effectively perform a linear extrapolation of historical trends. However, the advantage of the random walk with drift model lies in its utilization of longitudinal time series – incorporating multiple observation points – whereas the AiBi method relies exclusively on a bipunctual census-based approach. A further significant advantage of the random walk framework is that it enables the stochastic quantification of forecast uncertainty, yielding robust confidence intervals for future population estimates.

However, as illustrated in Figures 2 and 3, a primary disadvantage of the random walk method is that each municipality is projected autonomously, without the structural constraints imposed by the aggregate population of the overarching area. This lack of hierarchy can result in point estimates that deviate substantially from the expected aggregate total, a phenomenon that becomes increasingly pronounced over extended projection horizons.

To mitigate these discrepancies, we propose a methodological synthesis that harnesses the strengths of both approaches: utilizing the AiBi point projection to ensure geographic consistency, while incorporating the forecast uncertainty derived from the random walk with drift model.

Based on equation (6), the total forecast uncertainty (E_h) is defined by the integration of the accumulated stochastic variance and the parametric uncertainty of the linear trend. Consequently, we adopt the uncertainty intervals generated by the stochastic linear model and map them onto the AiBi point estimates. This integration is justified by the linear congruence between both methods, as both are predicated on the assumption of linear population change.

The forecast error (E_h), as derived from equation (6) using the historical population series of municipality i, is mapped onto the corresponding AiBi point projection as follows:

$$CV_i(t+h) = \frac{E_h}{Y_i\,(t+h)} \tag{7}$$

$$E_{h,AiBi} = CV_i(t+h)\hat{P}_i(t+h) \tag{8}$$

$$\hat{P}_i(t+h)_{interval} = \hat{P}_i(t+h) \pm z_{\propto/2}E_{h,AiBi} \tag{9}$$

Where:

$CV_i(t+h)$ is the coefficient of variation for period $t+h$ obtained from the random walk model;

E_h is the forecast error estimated from random walk with drift model (equation 6);

$Y_i(t+h)$ is the point estimate of the population in municipality i at time $t+h$ (from random walk with drift model);

$E_{h,AiBi}$ is the forecast error associated with the AiBi projection, predicated on the assumption that its relative uncertainty scales proportionally to the estimates derived from random walk model;

$\hat{P}_i(t+h)$ is the point estimate of the population of municipality i for period $t+h$ performed using the AiBi method;

$\hat{P}_i(t+h)_{interval}$ is the Interval forecast of the population of municipality i for the period $t+h$;

$z_{\propto/2}$ is the critical value of the normal distribution corresponding to the chosen confidence level.

Assuming an asymptotic normal distribution, for a 95% confidence interval, the integration of the random walk error term with AiBi point projection yields an interval forecast for each municipal projection. Thus, the 95% prediction interval for the population projection of municipality i, at an h steps-ahead horizon, is formulated as:

$$I_{forecast}95\%(t+h) = \textit{point forecast} \pm 1{,}96(E_{h,AiBi}) \tag{10}$$

Where:
$I_{forecast}95\%(t+h)$ represents the 95% prediction interval for the population forecast at horizon $(t + h)$, and "point forecast" refers to the point estimate for the period $(t + h)$, specifically, the estimator $\hat{P}_i(t+h)$ generated via the AiBi method.

If necessary, broader or narrower uncertainty bands can be tailored by adjusting the corresponding critical values. For instance, an 80% confidence interval would utilize a critical value of $Z_\alpha = 1.28$. The selection of an appropriate confidence level should be contingent upon the specific demographic dynamics of the area or the particular decision-making contexts, accounting for political, financial, or strategic planning involved.

The combination of the random walk with linear trend model and the AiBi point projection is particularly advantageous, as both frameworks are predicated on the assumption of linear trajectories for historical estimates and future forecasts. Consequently, the integration of robust uncertainty measures with point projections that maintain hierarchical consistency with the overarching projected area represents a significant methodological contribution of this research.

Furthermore, this hybrid framework demonstrates significant versatility, as it can be extended to integrate the forecast errors of alternative small-area projection methods. This adaptability establishes new pathways for broader applications of the proposed model within the field of stochastic demographic forecasting.

Results

The proposed model was empirically validated using population estimates and projections from the IBGE 2024 revision for the states of Rio Grande do Norte and São Paulo, spanning the horizon from 2000 to 2070. For the municipal estimates (or back projections) covering the 2000-2022 period, we utilized data from the 2000, 2010, and 2022 demographic censuses – adjusted for census undercount. The AiBi method was then applied to these figures to derive consistent intercensal estimates.

In total, the study encompasses 166[3] municipalities in Rio Grande do Norte and 645 municipalities in São Paulo. Annual projections were generated for each of these administrative units, disaggregated by sex covering the period from 2023 to 2070.

[3] The state of Rio Grande do Norte currently comprises 167 municipalities. However, the municipality of Jundiá was not listed in the 2000 Census, appearing in the state's official census records only from 2010 onward. To ensure longitudinal consistency for the analysis of population trends and future projections, Jundiá was aggregated with Várzea, the parent municipality from which it was dismembered. Consequently, the municipal database for Rio Grande do Norte employed in this study is based on the 166 administrative units existing as of 2000.

To facilitate data visualization, we present results for the female population across a representative selection of municipalities from both states. The model parameters of both AiBi and random walk with drift frameworks regarding these localities are detailed in the appendix tables. To ensure reproducibility, the complete dataset and the R script utilized for the analysis are available at https://demografiaufrn.net/projecao-populacional/.

Appendix 1 presents the model parameters, intercensal estimates, and projections derived via the AiBi method for the female populations of the selected municipalities in both states. For this backward projection – which functions as a linear interpolation between two censuses points – it was unnecessary to differentiate between growing and declining areas. Given that the interpolation is constrained by observed census counts, there is no risk of generating negative population values for declining municipalities, as the second census serves as the empirical boundary for the population trajectory.

However, when generating future projections using the AiBi method (2023-2070), we addressed the previously discussed limitations (1) and (2) by partitioning the municipalities of each state into two distinct groups. Based on the 2010 and 2022 census populations (adjusted for census undercount), the municipalities were categorized into: (a) those exhibiting intercensal population growth; and (b) those experiencing population decline.

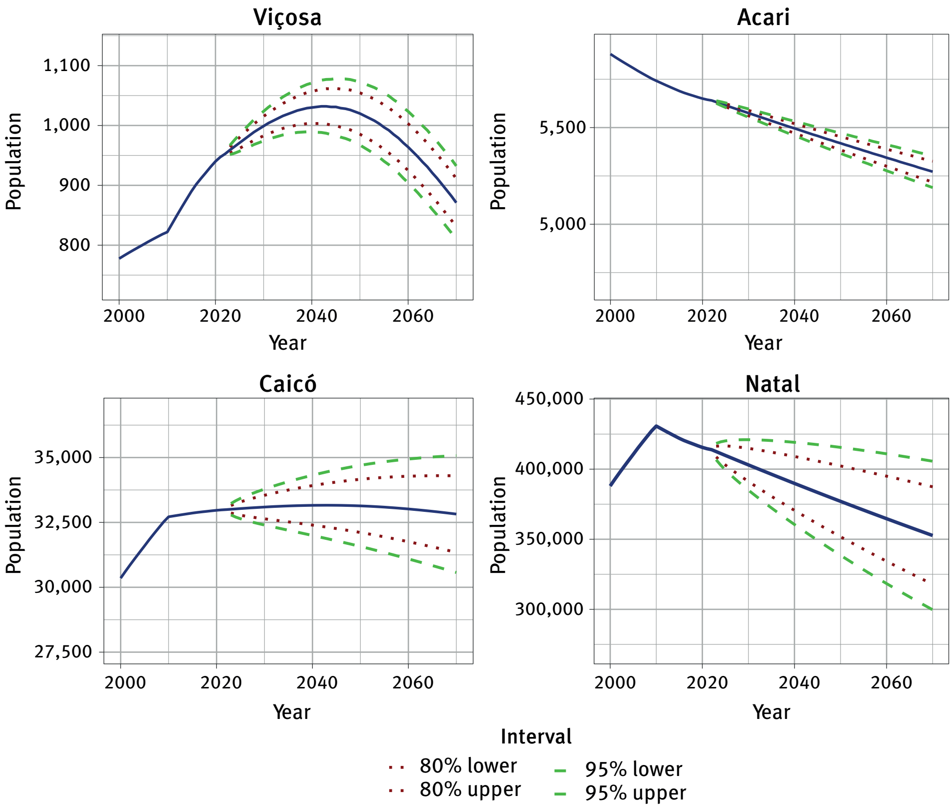
The total state population was partitioned accordingly into two sub-aggregates (growth and degrowth), ensuring that each municipal subgroup remained consistent with its corresponding component of the overarching. The procedure for projecting the population within these two aggregated subsets is detailed in the Methodology section. This bifurcation into two population groups serves as a non-negative constraint, preventing the occurrence of negative values in the later years of the projection horizon for municipalities characterized historical population decline.

As detailed in the Methodology section, the random walk with drift time series model utilized the historical series of municipal population data (2000 to 2022) to generate annual projections – disaggregated by sex – for the 2023-2070 horizon. The specific model parameters for the female populations of the four selected municipalities in each state are documented in Appendix 2.

Figure 4 illustrates the population trajectories for four municipalities in Rio Grande do Norte, selected to represent a diverse range of demographic profiles, including varying population sizes, growth dynamics, and geographic locations. Figure 5 provides analogous results for four municipalities in São Paulo.

These figures juxtapose the AiBi point projections with the 80% and 95% prediction intervals derived from equation (10), allowing for a direct comparison between the deterministic trend and the associated stochastic uncertainty.

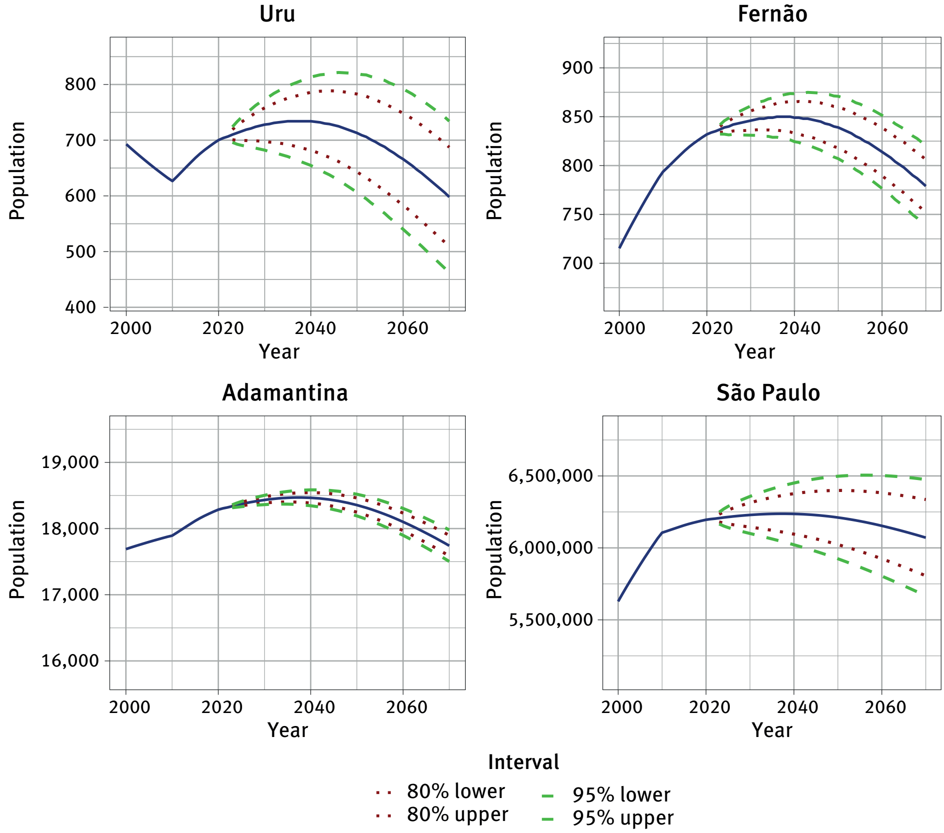
FIGURE 4
Female population trends
Selected municipalities in Rio Grande do Norte – 2000-2070



Source: IBGE. Brazilian Demographic Censuses 2000, 2010, and 2022. Census populations were corrected using IBGE's state-level projections, Revision 2024 (IBGE 2024b).

These results demonstrate a standard stochastic pattern: uncertainty scales proportionately with the projection horizon, as evidenced by the progressive widening of prediction intervals over time. Beyond the forecast duration, uncertainty is primarily driven by municipal population size and the stability of historical demographic trajectories. Consequently, the most pronounced forecast errors – quantified by the Coefficient of Variation (CV) in Appendix 2 and Figures 4–5 – and the broadest prediction intervals are concentrated in two types of administrative units: small municipalities with low exposure to risk (e.g., Viçosa, RN; Uru and Fernão, SP) and those experiencing significant structural shifts in growth direction, even among larger urban centers such as Natal (RN).

FIGURE 5
Female population trends
Selected municipalities in São Paulo – 2000-2070

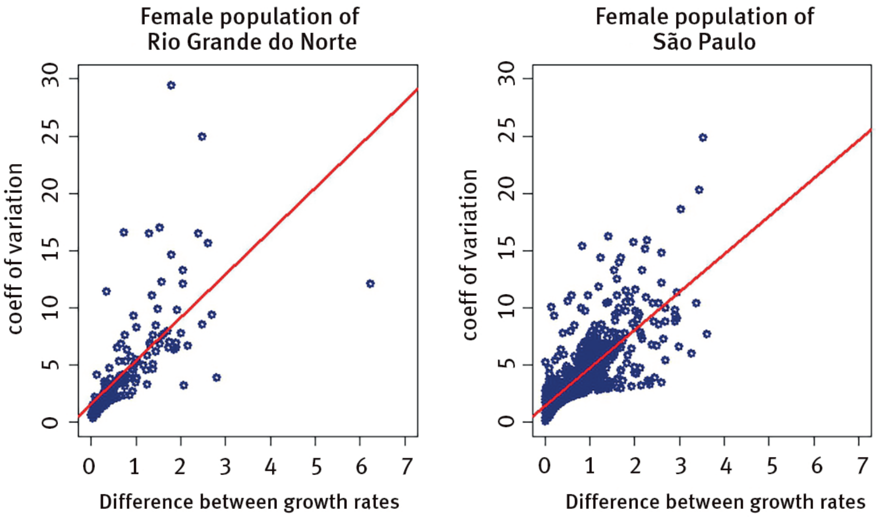


Source: IBGE. Brazilian Demographic Censuses 2000, 2010, and 2022. Census populations were corrected using IBGE's state-level projections, Revision 2024 (IBGE 2024b).

For instance, the municipality of Uru exhibits a shift in its historical demographic trend (Figure 5), transitioning from a decline between 1991 and 2000 to an increase between 2000 and 2010. Consequently, its prediction interval is significantly broader than that of Fernão, which, despite having a comparable population size, has maintained a consistently positive growth trajectory. This dynamic also accounts for the wider prediction intervals observed in Natal relative to Caicó. Although Caicó's female population is substantially smaller – approximately 25,000 women – than that of Natal (413,888 women, according to the 2022 census adjusted by IBGE's 2024 projections), Caicó's stable growth trend yields narrower prediction intervals. As illustrated in Figure 4, Natal's female population growth from 2000 to 2010 but entered a period of stagnation from 2010 to 2022. This shift in growth dynamics heightens forecast uncertainty for the upcoming decades. Conversely, Caicó maintained a consistently upward female population growth trajectory from 2000 to 2022. This greater historical stability is reflected in a narrower prediction interval compared to Natal, notwithstanding Caicó's substantially smaller population size.

This shift in growth dynamics further explains why several municipalities in São Paulo, despite their substantial population sizes, exhibit coefficients of variation that are higher than their scale would initially suggest. The anticipated inverse relationship between population size and forecast error is partially offset by historical growth volatility. Figure 6 illustrates this across all analyze municipalities in Rio Grande do Norte (RN) and São Paulo (SP), depicting the correlation between historical fluctuations in annual growth rates and the resulting degree of forecast uncertainty

FIGURE 6
Relationship between past fluctuations in population growth rate and the coefficient of variation of the 2070 projection



Source: IBGE. Brazilian Demographic Censuses 2000, 2010, and 2022. Census populations were corrected using IBGE's state-level projections, Revision 2024 (IBGE, 2024).

Figure 6 depicts the relationship between the absolute change in population growth rates across the 2000-2010 and 2010-2022 periods and the estimated coefficient of variation (CV) for each municipality by the terminal projection year (2070). A positive correlation is evident: municipalities experiencing more pronounced structural shifts in their historical growth rates exhibit heightened projection uncertainty at the forecast horizon. Consequently, forecast uncertainty – measured by the CV on the y-axis – scales proportionately with the instability of historical demographic patterns.

Final considerations

This study introduces an innovative methodological framework for small-area population projections by integrating the AiBi growth distribution method with stochastic uncertainty derived from a Random Walk with Drift (RWD) model. This hybrid approach ensures internal consistency between municipal and state-level population totals while enabling the derivation of prediction intervals. By doing so, it incorporates an explicit dimension of uncertainty into estimates that are conventionally presented as deterministic point values.

The findings indicate that forecast uncertainty scales with the projection horizon, aligning with established principles of stochastic theory. Furthermore, smaller municipalities exhibit broader prediction intervals, a reflection of the heightened volatility inherent in small-area demographic estimation. A critical insight from this analysis is that the stability of historical population trajectories is a primary determinant of forecast precision: jurisdictions characterized by consistent growth or decline yield more reliable estimates than those that have undergone significant structural shifts in the direction or magnitude of their population dynamics.

Utilizing female population counts from the 2022 Census (adjusted for under-enumeration according to the IBGE 2024 revision), pairwise comparisons –such as Acari (N = 5,640) *versus* Natal (N = 413,888) in Rio Grande do Norte, or Adamantina (N = 18,320) *versus* São Paulo (N = 6,203,115) – illustrate a key demographic principle: the stability of historical growth trajectories can effectively mitigate forecast uncertainty, even within relatively small populations.

These findings reinforce the practical utility of the proposed framework for both public and private sector strategic planning. This is particularly salient in contexts where minor fluctuations in population size can exert significant influence on resource allocation, infrastructure capacity, and the design of targeted social policies. By embedding stochastic uncertainty into projection outputs, the methodology provides a more robust and transparent decision-support tool. Ultimately, this approach enhances the reliability of small-area population forecasts, qualifying their application across a wide range operational setting.

Moreover, the proposed framework is highly adaptable and computationally straightforward, facilitating its replication across various administrative tiers and territorial scales. Through a bottom-up hierarchical aggregation – specifically, the summation of municipal-level outputs – it is possible to derive uncertainty estimates for higher geographic units, such as states. When aggregating AiBi point projections, the resulting state-level total remains perfectly consistent with independent IBGE state projection (as detailed in Table 1). Likewise, the summation of the lower and upper bounds of municipal projection intervals yields a comprehensive interval forecast for the state-level projection.

Looking ahead, future research should investigate the integration of alternative time-series models and hybrid combinations with other established small-area demographic techniques. Such advancements would significantly expand the methodological toolkit available to applied demography. While the Random Walk with Drift (RWD) model employed in this study assumes a linear trajectory – either consistently increasing or decreasing – this assumption does not necessarily undermine short-term projections for municipalities experiencing recent shifts. Nevertheless, further exploration into more flexible models, such as generalized ARIMA specifications or random walk models with segmented trends (structural breaks), could offer enhanced sensitivity to non-linear demographic transitions.

In conclusion, this study contributes to the advancement of the methodological toolkit in applied demography by proposing a framework that seamlessly integrates statistical consistency, transparency, and practical applicability.

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Resumo

Estimativas intervalares para projeções populacionais em pequenas áreas combinando o erro de previsão de um método de série temporal com o método da tendência do crescimento linear (AiBi)

O Censo Demográfico de 2022 revela uma transição demográfica avançada no Brasil, marcada por declínio da fecundidade, mudanças na estrutura etária e crescimento populacional mais lento. Entre 2010 e 2022, muitos municípios apresentaram redução no crescimento da população e alguns registraram mudança na tendência populacional. Essas alterações, combinadas com as disparidades econômicas, sociais e climáticas, aumentam a demanda por projeções populacionais municipais confiáveis. No Brasil, o método AiBi é comumente utilizado, mas sua precisão não é mensurada. Este estudo estima a incerteza das projeções municipais por sexo, integrando o método do passeio aleatório com tendência com as projeções pontuais do AiBi. A metodologia foi aplicada a municípios do Rio Grande do Norte e de São Paulo, que diferem na qualidade dos dados e nas condições socioeconômicas. Os resultados mostram que a incerteza varia com o tamanho da população, a estabilidade das tendências passadas e o horizonte de projeção, sendo maior em municípios menores, em áreas com padrões de crescimento voláteis e em projeções de longo prazo.

Palavras-chave: Projeções. Pequenas áreas. AiBi. Passeio aleatório. Intervalo de previsão

Resumen

Estimaciones de intervalo para proyecciones de población de áreas pequeñas: combinación del error de pronóstico de un enfoque de series temporales con el método de tendencia de crecimiento lineal (AiBi)

El Censo Demográfico de 2022 evidencia una transición demográfica avanzada en Brasil, caracterizada por la disminución de la fecundidad, las transformaciones en la estructura por edad y la desaceleración del crecimiento poblacional. Entre 2010 y 2022, numerosos municipios registraron una reducción del crecimiento, e incluso algunos revirtieron sus tendencias demográficas. Estos cambios, junto con las desigualdades económicas, sociales y climáticas, incrementan la demanda de proyecciones municipales confiables. En Brasil, el método AiBi es ampliamente utilizado, aunque su precisión no ha sido evaluada de manera sistemática. Este estudio tiene como objetivo estimar la incertidumbre de las proyecciones municipales por sexo, combinando las estimaciones puntuales del método AiBi con la incertidumbre derivada del modelo de caminata aleatoria con tendencia. La metodología se aplicó a los municipios de Rio Grande do Norte y São Paulo, que presentan diferencias en calidad de datos y contextos socioeconómicos. Los resultados muestran que la incertidumbre depende del tamaño poblacional, la estabilidad de las tendencias pasadas y el horizonte de proyección, siendo mayor en municipios pequeños, en aquellos con patrones de crecimiento más inestables y en proyecciones de largo plazo.

Palabras clave: Proyecciones. Áreas pequeñas. AiBi. Paseo aleatorio. Intervalo de predicción.

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APPENDIX 1

TABLE 1.1
Intercensal estimates and population projections using the AiBi method and model parameters estimated for each analysis period. Female population
Selected municipalities of Rio Grande do Norte – 2000-2070

Period / Year	Parameter	Municipality			
		Acari	Caicó	Natal	Viçosa
2000-2010	ai	-0.0008	0.0128	0.2319	0.0002
	bi	6976.7	11677.9	50426.4	427.3
2010-2022	ai	-0.0009	0.0023	-0.1422	0.0011
	bi	7143.8	28883.1	663925.4	-986.2
2022-2070	ai	0.0040	0.0019	0.6658	0.0009
	bi	3068.8	30843.7	-13743.5	-61.5
Estimated / projcted population					
2000		5,880	30,351	387,962	778
2003		5,835	31,117	401,807	792
2005		5,807	31,599	410,518	801
2007		5,779	32,076	419,149	810
2010		5,742	32,717	430,730	822
2013		5,709	32,805	425,381	863
2015		5,689	32,861	421,967	890
2017		5,672	32,907	419,169	912
2020		5,650	32,967	415,513	940
2025		5,616	33,034	409,778	971
2030		5,575	33,092	403,017	999
2035		5,535	33,135	396,362	1,019
2040		5,496	33,158	389,814	1,030
2045		5,457	33,159	383,371	1,031
2050		5,419	33,137	377,031	1,020
2055		5,381	33,089	370,791	997
2060		5,344	33,018	364,652	964
2065		5,308	32,928	358,610	921
2070		5,272	32,821	352,665	871

Source: Basic data are from the Brazilian Demographic Censuses of 2000, 2010, and 2022, conducted by IBGE. Projections of state populations from IBGE, 2024 revision, were also used to correct the census populations (IBGE, 2024b).

TABLE 1.2
Intercensal estimates and population projections using the AiBi method and model parameters estimated for each analysis period. Female population
Selected municipalities of São Paulo – 2000-2070

Period / Year	Parameter	Municipality			
		Uru	Fernão	Adamantina	São Paulo
2000-2010	ai	0.0000	0.0000	0.0001	0.2016
	bi	1236.6	70.2	16017.2	1701207.5
2010-2022	ai	0.0000	0.0000	0.0002	0.0563
	bi	-381.3	271.1	12504.8	4876317.1
2022-2070	ai	0.0000	0.0000	0.0002	0.0544
	bi	-263.4	332.2	13134.8	5020071.2
Estimated / projcted population					
2000		693	715	17,689	5,628,010
2003		671	741	17,756	5,784,498
2005		658	757	17,798	5,883,029
2007		645	773	17,838	5,976,928
2010		627	794	17,893	6,105,742
2013		652	807	18,027	6,136,432
2015		668	815	18,116	6,156,639
2017		683	823	18,192	6,173,967
2020		700	832	18,286	6,195,441
2025		716	840	18,370	6,214,740
2030		728	846	18,432	6,228,880
2035		734	849	18,465	6,236,328
2040		734	849	18,465	6,236,225
2045		727	846	18,428	6,227,946
2050		713	839	18,355	6,211,121
2055		692	828	18,244	6,185,990
2060		666	814	18,101	6,153,343
2065		634	798	17,931	6,114,575
2070		598	779	17,741	6,071,207

Source: Basic data are from the Brazilian Demographic Censuses of 2000, 2010, and 2022, conducted by IBGE. Projections of state populations from IBGE, 2024 revision, were also used to correct the census populations (IBGE, 2024b).

APPENDIX 2

TABLE 2.1
Estimates of the parameters of the random walk with drift model, for the projection of the female population
Selected municipalities in Rio Grande do Norte – 2025-2070

Year	Acari			Caicó			Natal			Viçosa		
	Mean	Eh	*CV%	Mean	Eh	*CV%	Mean	Eh	*CV%	Mean	Eh	*CV%
2025	5,608	6.09	0.11	33,354	202.62	0.61	417,424	5386.38	1.29	976	7.17	0.73
2030	5,553	10.90	0.20	33,955	362.46	1.07	423,316	9635.46	2.28	1,016	12.83	1.26
2035	5,498	15.01	0.27	34,556	499.06	1.44	429,209	13266.99	3.09	1,056	17.66	1.67
2040	5,444	18.88	0.35	35,156	627.79	1.79	435,101	16689.10	3.84	1,096	22.22	2.03
2045	5,389	22.63	0.42	35,757	752.70	2.11	440,994	2000.53	4.54	1,135	26.64	2.35
2050	5,335	26.32	0.49	36,358	875.41	2.41	446,886	23271.84	5.21	1,175	30.99	2.64
2055	5,280	29.97	0.57	36,958	996.75	2.70	452,778	26497.51	5.85	1,215	35.28	2.90
2060	5,225	33.59	0.64	37,559	1117.16	2.97	458,671	29698.47	6.47	1,255	39.54	3.15
2065	5,171	37.19	0.72	38,160	1236.92	3.24	464,563	32881.95	7.08	1,295	43.78	3.38
2070	5,116	40.78	0.80	38,760	1356.18	3.50	470,456	36052.58	7.66	1,334	48.00	3.60

Source: Basic data are from the Brazilian Demographic Censuses of 2000, 2010, and 2022, conducted by IBGE. Projections of state populations from IBGE, 2024 revision, were also used to correct the census populations (IBGE, 2024b).

TABLE 2.2
Estimates of the parameters of the random walk with drift model, for the projection of the female population
Selected municipalities in São Paulo – 2025-2070

Year	Uru			Fernão			Adamantina			São Paulo		
	Mean	Eh	*CV%	Mean	Eh	*CV%	Mean	Eh	*CV%	Mean	Eh	*CV%
2025	708	12.78	1.80	852	4.44	0.52	18,405	20.16	0.11	6,281,539	37805.83	0.60
2030	711	22.86	3.21	879	7.94	0.90	18,549	36.06	0.19	6.412,244	67629.13	1.05
2035	715	31.47	4.40	906	10.93	1.21	18,692	49.65	0.27	6,542.950	93118.05	1.42
2040	718	39.59	5.52	933	13.75	1.47	18,835	62.46	0.33	6,673.655	117137.09	1.76
2045	721	47.47	6.59	961	16.49	1.72	18,978	74.88	0.39	6,804,361	140442.44	2.06
2050	724	55.21	7.63	988	19.17	1.94	19,122	87.09	0.46	6,935,067	163339.87	2.36
2055	727	62.86	8.65	1,015	21.83	2.15	19,265	99.16	0.51	7,065,772	185980.11	2.63
2060	730	70.45	9.65	1,042	24.47	2.35	19,408	111.14	0.57	7.196,478	208446.98	2.90
2065	733	78.00	10.64	1,070	27.09	2.53	19,551	123.05	0.63	7,327,184	230791.11	3.15
2070	736	85.52	11.61	1,097	29.70	2.71	19,694	134.92	0.69	7,457,889	253045.03	3.39

Source: Basic data are from the Brazilian Demographic Censuses of 2000, 2010, and 2022, conducted by IBGE. Projections of state populations from IBGE, 2024 revision, were also used to correct the census populations (IBGE, 2024b).
*Coefficient of variation.